Parameterization of Lakes in Numerical Models for Environmental Applications

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1 The Problem

Lakes significantly affect the structure of the atmospheric surface layer and therefore the surface fluxes of heat, water vapour and momentum. This effect has not been systematically studied so far and is poorly understood. In most numerical weather prediction (NWP), climate modelling and other numerical prediction systems for environmental applications, the effect of lakes is either entirely ignored or is parameterized very crudely. A physically sound model is required to predict the lake surface temperature and the effect of lakes on the structure and transport properties of the atmospheric surface layer. Apart from being physically sound, a lake model must meet stringent requirements of computational economy.

The problem is twofold. For one thing, the interaction of the atmosphere with the underlying surface is strongly dependent on the surface temperature and its time-rate-of-change. It is common for NWP systems to assume that the water surface temperature can be kept constant over the forecast period. The assumption is to some extent justified for seas and deep lakes. It is doubtful for small-to-medium size relatively shallow lakes, where the short-term variations of the surface temperature (with a period of several hours to one day) reach several degrees. At present, a large number of such lakes are indistinguishable sub-grid scale features. These lakes will become resolved scale features as the horizontal resolution is increased. In numerical prediction systems with coarser resolution, many small-to-medium size lakes remain sub-grid scale features. However, the presence of these lakes cannot be ignored due to their aggregate effect on the grid-scale surface fluxes.

Another important aspect of the problem is that lakes strongly modify the structure and the transport properties of the atmospheric surface layer. A major outstanding question is the parameterization of the roughness of the water surface with respect to wind and to scalar quantities, such as potential temperature and specific humidity. This second aspect of the problem is beyond the scope of the present paper.

In the present paper, a lake model capable of predicting the surface temperature in lakes of various depths on time scales from a few hours to a year is presented. The model is based on a two-layer parameterization of the temperature profile, where the structure of the stratified layer between the upper mixed layer and the basin bottom, the lake thermocline, is described using the concept of self-similarity of the evolving temperature profile. The same concept is used to describe the interaction of the water column with bottom sediments and the evolution of the ice and snow cover. This approach, that is based on what could be called "verifiable empiricism" but

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still incorporates much of the essential physics, offers a very good compromise between physical realism and computational economy.

2 Basic Concept

The concept of self-similarity of the temperature profile $T(z, t)$ in the thermocline was put forward by Kitaigorodskii and Miropolsky (1970) to describe the vertical temperature structure of the oceanic seasonal thermocline. The essence of the concept is that the dimensionless temperature profile in the thermocline can be fairly accurately parameterized through a universal function of dimensionless depth, that is

$$\frac{T_m(t) - T(z, t)}{\Delta T(t)} = \Phi_T(\zeta) \quad \text{at} \;\; h(t) \le z \le h(t) + \Delta h(t). \tag{1}$$

Here, t is time, z is depth, $T_m(t)$ is the temperature of the upper mixed layer of depth $h(t)$, $\Delta T(t) = T_m(t) - T_b(t)$ is the temperature difference across the thermocline of depth $\Delta h(t)$, $T_b(t)$ is the temperature at the bottom of the thermocline, and $\Phi_T \equiv [T_m(t) - T(z, t)] / \Delta T(t)$ is a dimensionless function of dimensionless depth $\zeta \equiv [z - h(t)] / \Delta h(t)$ that satisfies the boundary conditions $\Phi_T(0) = 0$ and $\Phi_T(1) = 1$. The temperature profile given by Eq. (1) is illustrated in Fig. 1.

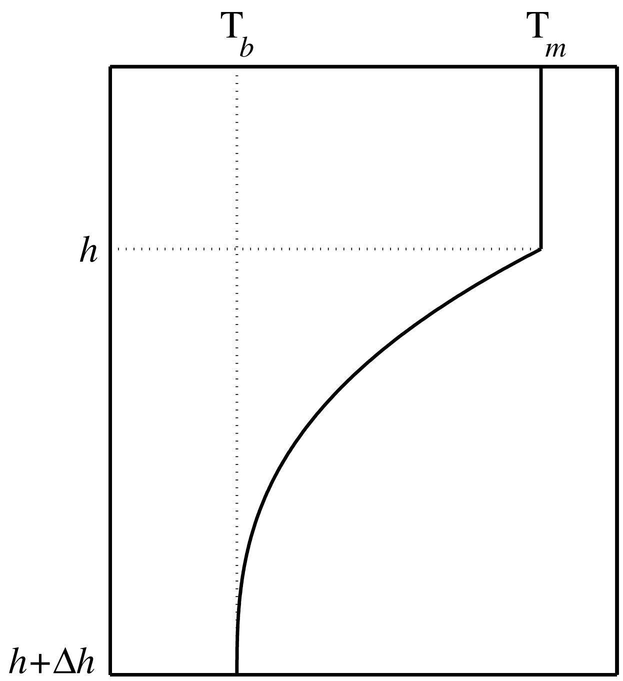

Figure 1: Schematic representation of the temperature profile in the upper mixed layer and in the thermocline. See text for notation.

The idea of self-similarity of the temperature profile in the thermocline can be traced back to the famous work of Munk and Anderson (1948). The following quotation (Munk and Anderson 1948, p. 276) is a qualitative statement of the idea: "…the upper layers are stirred until an almost homogeneous layer is formed, bounded beneath by a region of marked temperature gradient, the thermocline. …If the wind increases in intensity the thermocline moves downward, but the characteristic *shape* of the temperature-depth curve remains essentially unchanged." (Original authors' italic.)

The concept of self-similarity of the temperature profile in the thermocline can be viewed as a natural extension of the concept of the temperature uniform mixed layer that has been successfully used in geophysical fluid dynamics over several decades. Using the mixed-layer temperature $T_m(t)$ and its depth $h(t)$ as appropriate scales, the mixed-layer concept is expressed as $T(z,t)/T_m(t) = \vartheta[z/h(t)]$, where a dimensionless function ϑ is simply a constant equal to one. The use of ΔT and Δh as appropriate scales of temperature and depth, respectively, in the thermocline leads to Eq. (1), where Φ_T is not merely a constant but a more sophisticated function of ζ.

The concept of self-similarity of the temperature profile in the thermocline received support through laboratory (e.g. Linden 1975, Wyatt 1978) and observational (e.g. Miropolsky et al. 1970, Efimov and Tsarenko 1980, Filyushkin and Miropolsky 1981, Mälkki and Tamsalu 1985, Zilitinkevich 1991) studies. A plausible theoretical explanation for the observed self-similarity of the temperature profile in the thermocline was offered in the case of mixed-layer deepening, $dh/dt > 0$ (e.g. Barenblatt 1978, Turner 1978, Zilitinkevich et al. 1988, Zilitinkevich and Mironov 1992). Introducing a vertical co-ordinate moving with the mixed layer-thermocline interface and invoking one or the other closure model to specify the eddy heat conductivity in the thermocline, these authors obtained travelling-wave type solutions to the heat transfer equation. These solutions proved to be fairly similar to empirical polynomial approximations of the $\Phi_T(\zeta)$ curve obtained on the basis of empirical data. In the case of mixed-layer stationary state or retreat, $dh/dt \leq 0$, no theoretical explanation for the self-similarity of the temperature profile has been offered so far. The self-similarity at $dh/dt \leq 0$ is based on empirical evidence only and should therefore be considered phenomenological.

A distinctive feature of shallow lakes is a strong thermal interaction between the water body and the bottom sediments. A sizable portion of the heat received from the atmosphere during spring and summer can be accumulated in the thermally active upper layer of bottom sediments. This heat is then returned back to the water column during autumn and winter, leading to a hysteresis-like behaviour of the seasonal temperature cycle of the water column-bottom sediment system. A straightforward approach to describe the evolution of the thermal structure of bottom sediments is to use the equation of heat transfer with a priori knowledge of the thermal diffusivity of sediments. A shortcoming of this straightforward approach is that the thermal diffusivity is strongly dependent on the composition of the sediments and is, therefore, rarely well known. Golosov and Kreiman (1992) proposed an alternative way of describing the vertical temperature structure of bottom sediments. It is based on a two-layer parametric representation of the temperature profile in the bottom sediments that is conceptually similar to a parametric representation of the temperature profile in the thermocline. Then, the use of the integral heat budget of the thermally active layer of bottom sediments yields a model where the thermal diffusivity of sediments is no longer needed.

Many lakes are frozen over a considerable part of the year so that the atmosphere does not directly communicate with the lake water. The atmosphere-lake interaction occurs through the air-ice or, if snow is present, through the air-snow interface. An ice-snow model is therefore required to predict the surface temperature. Use of sophisticated ice models with rheology is a standard practice in climate modelling where the integration is performed over many decades. For NWP and related applications, a sophisticated dynamic-thermodynamic ice model is not required (and most often cannot be afforded because of the high computation cost). A simplified thermodynamic model is usually sufficient. Again, the approach based on a parametric representation (assumed shape) of the temperature profile within ice and snow and on the integral heat budgets of the ice and snow layers offers a good compromise between physical realism and computational economy.

Notice that the assumption about the shape of the temperature profile within the ice, the simplest of which is the linear profile, is either explicit or implicit in many ice models developed to date. A model of ice growth based on a linear temperature distribution was proposed by Stefan as early as 1891.

The concept of self-similarity of the evolving temperature profile has found use in modelling geophysical flows. Computationally-efficient models have been developed and successfully applied to simulate the evolution of the mixed layer and seasonal thermocline in the ocean (e.g. Kitaigorodskii and Miropolsky 1970, Arsenyev and Felzenbaum 1977, Filyushkin and Miropolsky 1981) and of the atmospheric convectively mixed layer capped by a temperature inversion (e.g. Deardorff 1979, Fedorovich and Mironov 1995). Models of the seasonal cycle of temperature and mixing in medium-depth fresh-water lakes, based on the self-similar representation of the evolving temperature profile, have been developed and successfully applied by Zilitinkevich and Rumyantsev (1990), Zilitinkevich (1991), Mironov et al. (1991) and Golosov et al. (1998). A first attempt has been made to apply the above self-similarity concept to shallow lakes and to consider short-term (diurnal) variations of temperature and mixing conditions (Kirillin 2001). As different from the ocean and the atmosphere, where the thermocline (capping inversion) is underlain (overlain) by a deep stably or neutrally stratified quiescent layer, the above lake models assume a two-layer temperature structure, where the thermocline extends from the bottom of the mixed layer down to the basin bottom. This assumption is fair for most lakes, except for very deep lakes such as Lake Baikal.

3 The Lake Model

In this section, we present a very brief description of a lake model based on a self-similar representation of the temperature profile in the water column, in the bottom sediments and in the ice and snow (a detailed description of the model is given by Mironov 2003). The same basic concept is used to describe the temperature structure of the four media in question (snow, ice, water and sediment). The result is a computationally efficient lake model that incorporates much of the essential physics. The model proposed by Mironov et al. (1991) was taken as a starting point.

The lake model is based on the two-layer parameterization of the vertical temperature profile given by Eq. (1), where the thermocline is assumed to extend from the outer edge of the mixed layer, $z = h$, down to the lake bottom, $z = h + \Delta h = D$. According to Eq. (1), the evolving temperature profile is characterized by three time-dependent parameters, namely, the mixed-layer depth $h(t)$, its temperature $T_m(t)$, and the bottom temperature $T_b(t)$. These quantities are related to each other and to the mean temperature of the water column, $\overline{T}(t) \equiv D^{-1} \int_0^D T(z,t)dz$, through $\overline{T} = T_m - C_T(1 - h/D)(T_m - T_b)$. The quantity $C_T = \int_0^1 \Phi_T(\zeta)d\zeta$ is the so-called shape factor. We notice at once that, although the function $\Phi_T(\zeta)$ is useful in that it provides a continuous temperature profile through the water column, its exact shape is not required in our model. It is not Φ_T per se, but the shape factor C_T that enters the resulting equations.

The evolution equations for the above time-dependent quantities are developed by using the integral, or bulk, approach. The volumetric character of the short-wave radiation heating is accounted for. Integrating the heat transfer equation over the water column, i.e. from $z = 0$ to $z = D$, yields the equation for the mean temperature $\overline{T}(t)$. Integrating the heat transfer equation over the mixed layer, i.e. from $z = 0$ to $z = h$, yields the equation for the mixed-layer temperature $T_m(t)$. The equation for the bottom temperature depends on the mixed layer state. During the mixed layer stationary state or retreat, $dh/dt \leq 0$, the bottom temperature is kept unchanged, $dT_b/dt = 0$. During the mixed layer deepening, $dh/dt > 0$, the equation for $T_b(t)$ is obtained by means of double integration of the equation of heat transfer over the thermocline with due regard for the parameterization (1) (i.e. the integration is first performed over z' from h to $z > h$, and the result is then integrated over z from h to D). To this end, we use the idea of Filyushkin and Miropolsky (1981) that, in case of the mixed layer deepening, not only the profile of temperature but also the profile of the vertical turbulent heat flux in the thermocline can be represented in a self-similar form. This idea has received support through observational studies (Filyushkin and Miropolsky 1981, Tamsalu et al. 1997). Furthermore, the self-similarity of the vertical profile of the heat flux is suggested by an analytical travelling-wave type solution to the heat transfer equation.

The evolution equation for the mixed-layer depth is developed in a usual way, on the basis of the turbulence kinetic energy equation integrated over the mixed layer. Convective deepening of the mixed layer is described by the entrainment equation. It incorporates the Zilitinkevich (1975) spin-up correction term that prevents an unduly fast growth of $h(t)$ when the mixed-layer is shallow. The entrainment equation is also capable of predicting the equilibrium depth of a convectively mixed layer, where convective motions are driven by surface cooling, whereas the volumetric radiation heating tends to arrest the mixed layer deepening (Mironov and Karlin 1989). The depth of the wind-mixed layer is computed from a relaxation-type equation, where a multi-limit boundary-layer formulation proposed by Zilitinkevich and Mironov (1996) is used to compute the equilibrium mixed-layer depth.

A two-layer parameterization proposed by Golosov and Kreiman (1992) and further developed by Golosov et al. (1998) is used to predict the vertical structure of the thermally-active upper layer of bottom sediments and the heat flux through the water-sediment interface. Observations suggest that the temperature profile in the bottom sediments has the form of a travelling thermal wave. The wave starts at the water-sediment interface and propagates downward as the lake water and the bottom sediments are heated during spring and summer. When heating ceases and cooling sets in, a new wave starts at the water-sediment interface. It propagates downward as the lake water and the sediments are cooled during autumn and winter, thus closing the annual cycle. Importantly, a characteristic *shape* of the temperature-depth curve remains approximately the same. Using a two-layer parameterization proposed by Golosov et al. (1998), the evolving temperature profile in the sediments is described by two time-dependent parameters, namely, the depth $H_b(t)$ penetrated by the thermal wave and the temperature $T_H(t)$ at this depth. The evolution equations for these quantities are derived by integrating the heat transfer equation over z from the lake bottom $z = D$ to the depth $z = H_b$ penetrated by the thermal wave, and from $z = H_b$ to the depth $z = L$ of the thermally active layer of bottom sediments (at $z > L$, the seasonal temperature changes are negligible). The problem is closed by the diagnostic relation for the bottom heat flux. Importantly, the resulting equations do not incorporate the thermal diffusivity of sediments, a quantity that is rarely known to a satisfactory degree of precision.

An ice-snow model is developed, using a parametric representation (assumed shape) of the evolving temperature profile within ice and snow. That is, the basic concept is the same as the concept of self-similarity of the thermocline. Using the assumed shape of the temperature profile, the heat transfer equation is integrated over depth from the lower side to the upper side of the ice to yield the equation of the heat budget of the ice layer. The evolution equation for the ice thickness is developed by considering the heat budget of the ice slab with due regard for the heat release/consumption caused by the ice accretion/ablation. The result is an ice model that consists of two ordinary differential equations for the two time-dependent quantities, namely, the temperature $T_i(t)$ at the upper side of the ice and the ice thickness $H_i(t)$. The snow model is developed in a similar way except that the rate of snow accumulation is not computed within the snow model but is assumed to be a known time-dependent quantity that is provided by the driving atmospheric model or is known from observations. The result is two ordinary differential equations for the two time-dependent quantities, namely, the temperature $T_s(t)$ at the upper side of the snow and the snow thickness $H_s(t)$.

Finally, we end up with a system of ordinary differential equations for several time-dependent quantities. These are the mean temperature of the water column, the mixed-layer temperature and its depth, the temperature at the lake bottom, the temperature at the bottom of the upper layer of bottom sediments penetrated by the thermal wave, and the depth of this layer. In case the lake is cover by ice and snow, four additional quantities are computed, namely, the temperatures at the air-snow and snow-ice interfaces, the snow depth and the ice depth. This system of differential equations is supplemented by a number of algebraic (or transcendental) equations for diagnostic quantities, such as the heat flux through the lake bottom and the heat flux through the ice-snow interface. Optionally, some modules can be switched off, e.g. the snow module or the bottom-sediment module. The lake model includes a number of thermodynamic parameters. These are taken to be constant except for the snow density and the snow heat conductivity that are functions of the snow depth.

The lake model described above contains a number of dimensionless constants and empirical parameters. Most of them are estimated with a fair degree of confidence. It must be emphasised that the empirical constants and parameters of the lake model are not application-specific. That is, once they have been estimated, using independent empirical and numerical data, they should not be re-evaluated when the model is applied to a particular lake. In this way we avoid "re-tuning" of the model, a procedure that may improve an agreement with a limited amount of data and is sometimes justified. This procedure should, however, be considered as a bad practice and must be avoided whenever possible as it greatly reduces the predictive capacity of a physical model (Randall and Wielicki 1997).

Apart from the optical characteristics of lake water, the only lake-specific parameters are the lake depth, the depth of the thermally active layer of bottom sediments and the temperature at this depth. These parameters should be estimated only once for each lake, using observational data or empirical recipes. In a similar way, the temperature at the bottom of the thermally active soil layer and the depth of this layer are estimated once and then used in an NWP system as two-dimensional external parameter arrays.

4 First Results

The lake model described above is being extensively tested and further developed. First results are exemplified by Figs. 2 and 3. These figures compare the modeled and measured mixed-layer temperature, bottom temperature and mixed-layer depth in Müggelsee ("See" is German for "lake"), a fresh-water lake located near Berlin, Germany. The lake covers an area of 7.3 km^2. Its average depth is 4.8 m, and the maximum depth is 8 m. The thermal regime of Müggelsee is characterized by a pronounced seasonal temperature cycle. In spite of a comparatively low depth of the lake, it is not always mixed down to the bottom. The thermocline exists over appreciable lengths of time from late spring through early autumn.

The temperature measurements shown in Figs. 2 and 3 are taken at the station where the depth to the bottom is $D = 7$ m. The lake model is forced by the observed incident radiation flux. The upward long-wave radiation flux is computed through the Stefan-Boltzmann law. The fluxes of sensible and latent heat are computed, using data from the surface-layer meteorological measurements and the atmospheric surface-layer parameterization scheme described in Mironov (1991). The model time step is 24 h.

As seen from Figs. 2 and 3, the simulated mixed-layer temperature and bottom temperature show an overall satisfactory agreement with observations. The model tends to slightly overestimate the bottom temperature, suggesting that the lake thermocline is somewhat too diffusive. Work is

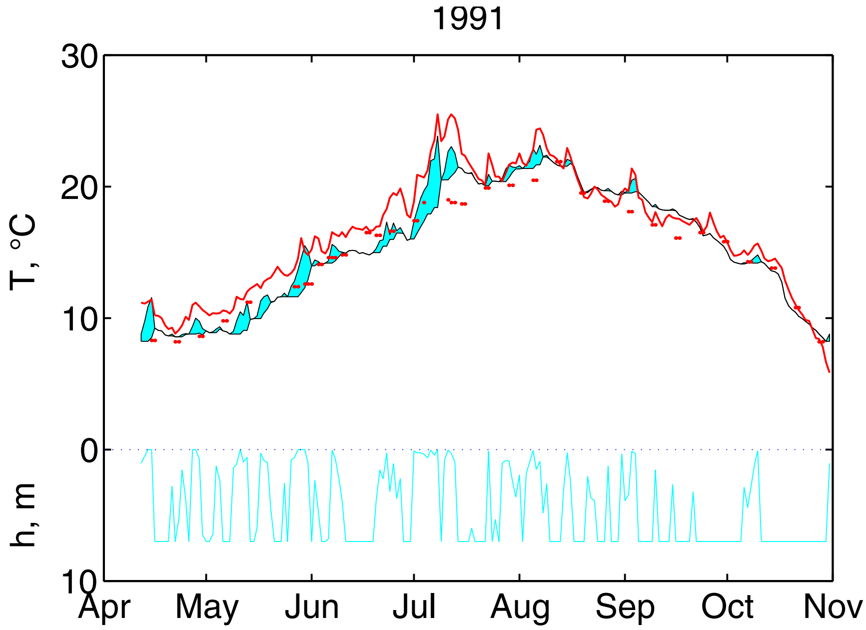

Figure 2: The evolution of the temperature structure in Müggelsee during April – November 1991. Heavy solid curve shows the observed mixed-layer temperature. Dots show the observed bottom temperature. Thin solid curves show the mixed-layer temperature (upper curve) and the bottom temperature (lower curve) computed with the lake model. Shaded areas mark the periods of stable density stratification in the lake thermocline, i.e. when the lake is not mixed down to the bottom. The computed mixed-layer depth is shown by thin solid line in the lower part of the plot.

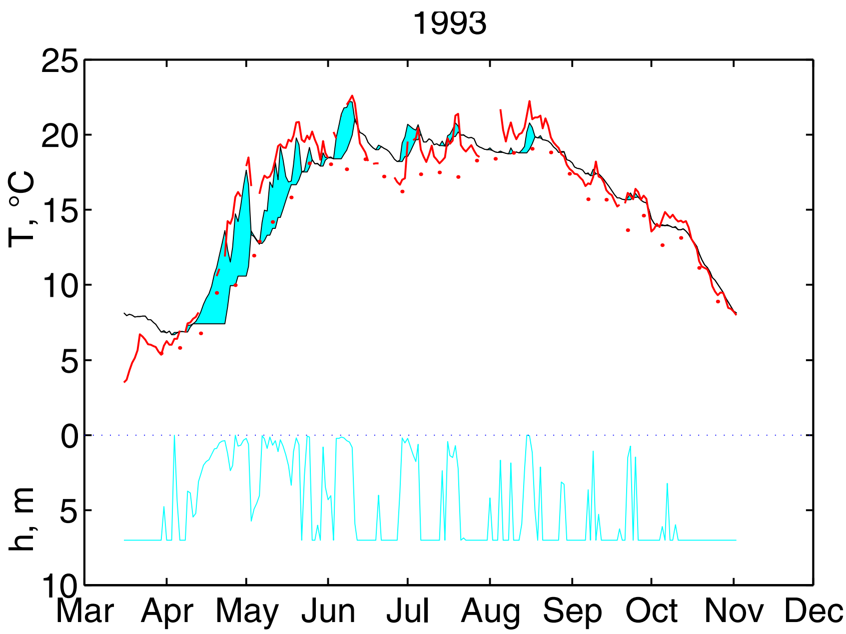

Figure 3: The same as in Fig. 2, but for the period March – December 1993.

underway to cure the trouble. Further numerical experiments are performed to test the ability of the proposed lake model to simulate short-term (diurnal) variations of the temperature structure.

5 Conclusions

We have presented a lake model suitable to predict the vertical temperature structure in lakes of various depth. The model is based on a two-layer parameterization of the temperature profile, where the structure of the stratified layer between the upper mixed layer and the basin bottom, the lake thermocline, is described using the concept of self-similarity of the evolving temperature profile. The same concept is used to describe the interaction of the water column with bottom sediments and the evolution of the ice and snow cover.

The proposed lake model is intended for use, first of all, in the NWP and climate modelling systems as a module to predict the water surface temperature. Apart from NWP and climate modelling, practical applications, where simple parameterized models are favoured over more accurate but more sophisticated models (e.g. second-order turbulence closures), include modelling aquatic ecosystems. For ecological modelling, a sophisticated physical module is most often not required because of insufficient knowledge of chemistry and biology.

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