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Spring Convection in Ice-Covered Freshwater Lakes

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Abstract—The regime of penetrative convection driven by a vertically inhomogeneous radiation heating is considered. Similar convection is observed in ice-covered freshwater lakes, where the water temperature is below the maximum-density temperature during late spring. The convective length, velocity, and temperature scales are introduced, which are suitable for the description of convection driven by a vertically inhomogeneous heating. On the basis of these scales and the similarity hypotheses for the vertical temperature profiles and turbulence characteristics in the convective mixed layer, a simple model of the convection regime under consideration is proposed. The entrainment regimes characteristic of the convection in ice-covered lakes are studied. It is shown that such convection is satisfactorily described by the equation used for convection in both the atmospheric and oceanic layers if the Deardorff convective scales based on the buoyancy flow through a fluid surface are replaced by the convective scales, which take into account the three-dimensional character of radiation absorption. The results of model calculations are in good agreement with observational data.

INTRODUCTION

Observations in ice-covered lakes show that in late spring the bulk of water is vertically well mixed and characterized by an almost constant temperature with depth [1–4]. It is found that the mixing is caused by convection driven by a vertically inhomogeneous heating due to absorption of solar radiation penetrating into the water beneath the ice. At a water temperature above the temperature of maximum density, convection in the upper oceanic layer (see, for example, [5]) and in freshwater lakes (see, for example, [6]) also depends on the intensity of solar heating and the character of solar radiation absorption. However, when the water temperature is higher than the temperature of maximum density, the source of convective motions is a surface cooling, while radiation heating increases the static stability of water column and, thus, hinders the mixed layer deepening. At a temperature below the temperature of maximum density, the regime of convection in freshwater lakes has a quite different character. Under this regime, solar heating results in an unstable water column and, thus, initiates convective motions. This work is devoted to this regime of convection. On the basis of observational data and some common physical considerations, the convective length, velocity, and temperature scales are used to describe the convection regime under consideration. On the basis of the introduced scales and the similarity hypotheses for the vertical profiles of temperature and turbulence characteristics, a simple parametrized model of a convective mixed layer is proposed. The calculation results obtained using the proposed model are compared to the observational data. The entrainment regimes characteristic of convection in ice-covered lakes are analyzed. Different versions of the entrainment equation to describe the type of convection under study are discussed (see [1, 2, 7]).

The first systematic observations of spring convection in an ice-covered freshwater lake were carried out by Farmer [1]. Water temperature was measured with a bathythermograph and the thermistor chains in Lake Beibin in the western part of Canada from February 1 to April 15, 1973, which allowed Farmer to follow the occurrence and development of a convective mixed layer. Farmer was apparently the first who analyzed theoretically the development of convective instability in the ice-covered lake due to a vertically inhomogeneous radiation heating and proposed a highly reliable model of the convection regime under consideration.

Petrov and Sutyrin considered the diurnal cycle of convection beneath the ice [2]. Using the data of observations and a simple model of the phenomenon, they analyzed some regimes of convective mixed layer deepening, which were realized at different times of the day.

The detailed observations of convection beneath the ice were carried out in Lake Vendyurskoe, Karelia, in the spring of 1995 [3, 4, 8, 9]. Compared to the field investigations performed earlier, the temperature was measured with a higher resolution along the vertical. In particular, detailed data were obtained on temperature distribution in a thin stably stratified layer separating

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the convective mixed layer from the ice undersurface. Moreover, direct measurements of solar radiation reaching the ice undersurface were taken.

The accumulated empirical data imply the following qualitative picture of the phenomenon. In winter and early spring, the snow that covers the ice hinders the penetration of solar radiation into the water through the ice. Variations in the water temperature are due to molecular heat transfer, and, therefore, they are very slow. As the snow is melting, solar radiation begins to penetrate through the ice into the water. Radiation heating of the water column is inhomogeneous along the vertical: the upper layers receive more heat than the lower ones. As a result of heating, a portion of the water column becomes hydrostatically unstable, and convection occurs. The convection forms a well-mixed layer with a constant or almost constant temperature in depth. A rapid temperature difference within a thin layer on the lower boundary of the mixed layer shows that convection is penetrative.

An analysis of the temperature profiles within a period of penetrative convection (see, for example, Fig. 16 in [3]) allows one to distinguish four layers. Within a relatively thin surface layer just beneath the ice, the water temperature increases rapidly from the freezing temperature at the water–ice interface to the value characteristic of the bulk of the mixed layer. Convective motions in the mixed layer provide its temperature uniformity along the vertical. The thermocline characterized by a rapid increase of temperature with depth is formed on the lower boundary of the mixed layer. This thermocline can be identified with a layer of turbulent entrainment, in which the lower denser water is entrained by turbulent disturbances. Below this layer, there is a stably stratified undisturbed layer, in which temperature variations are due to solar radiation absorption and molecular heat transfer.

Both the depth and temperature of the mixed layer increase with heating. It is very important that the evolving temperature profile holds its four-layered structure. This makes it possible to use a self-similar parametric representation of the temperature profile within a period of penetrative convection.

HEAT BUDGET

Let us use the quadratic equation of the freshwater state:

$$\rho = \rho_r\left[1 - \frac{1}{2}a_T(\theta - \theta_r)^2\right], \tag{1}$$

where ρ is the water density, θ is the water temperature, $\rho_r \approx 999.8$ kg m^{-3} is the maximum freshwater density corresponding to the temperature $\theta_r \approx 277$ K, and $a_T = 1.65 \times 10^{-5}$ K^{-2} is the empirical coefficient [10].

Let us take the following representation of the vertical temperature profile within the period of penetrative convection:

$$\theta = \begin{cases} \theta_s \text{ for } 0 \le z \le \delta, \\ \theta_m \text{ for } \delta \le z < h, \\ \theta_q \text{ for } h < z \le D. \end{cases} \tag{2}$$

Here, z is the depth; t is the time; $\delta(t)$ is the depth of the surface layer beneath the ice; $\theta_s(t, z)$ is the water temperature in the surface layer; $h(t)$ is the depth of the lower boundary of the convective mixed layer (CML); $\theta_m(t)$ is the water temperature for the CML; D is the locality depth; and $\theta_q(t, z)$ is the water temperature in the undisturbed layer, which is below the entrainment layer. Within the framework of the idealized representation (2), the entrainment layer depth is zero, and the temperature profile is approximated by the temperature abrupt change $\Delta\theta = \theta_q(h) - \theta_m$.

The water temperature in the surface layer beneath the ice increases rapidly with depth from the freezing temperature $\theta_f \approx 273$ K at the water–ice interface to the mixed layer temperature θ_m (see, for example, Fig. 15 in [3]). Turbulence in the surface layer is suppressed by the stable density stratification, and the temperature distribution in this region is described by the heat conduction equation $\partial\theta/\partial t = k\partial^2\theta/\partial z^2 - \partial I/\partial z$, where $k \approx 1.3 \times 10^{-7}$ m^2 s^{-1} is the molecular thermal water conductivity, $I(t, z)$ is the kinematic solar radiation flux (i.e., the radiation flux divided by the water density ρ and the water specific heat at a constant pressure $c_p \approx 4.2 \times 10^3$ J kg^{-1} K^{-1}). In accordance with a rough character of our model, we will not consider the properties of the transition zone between the turbulized mixed layer and the nonturbulized layer immediately beneath the lower ice surface, and we shall extend the “molecular” solution up to the upper boundary of the mixed layer. Let us assume that the regime of heat transfer in the surface layer may be treated as quasi-stationary. At $\partial\theta/\partial t = 0$, the solution of the heat conduction equation given above has the following form:

$$\theta_s = \theta_f + \frac{z}{\delta}(\theta_m - \theta_f) + k^{-1}\left(\int_0^z I dz' - \frac{z}{\delta}\int_0^\delta I dz\right). \tag{3}$$

The depth of the surface layer δ is determined from the condition of a smooth joining of the temperature profile on the lower boundary of the surface layer, $\partial\theta/\partial z = 0$ at $z = \delta$. The described parametrization of the surface layer was proposed by Barnes and Hobbie [11].

In the undisturbed region, which is below the entrainment layer, the water temperature is determined from the heat conduction equation. Neglecting the molecular heat transfer, we obtain the following expression:

$$\theta_q = \theta_{ini} + \int_0^t (-\partial I/\partial z)dt', \tag{4}$$

where $\theta_{ini}(z)$ is the initial temperature profile.

In the mixed layer, the molecular heat flux is equal to zero. Here, the heat transfer equation has the form $\partial\theta/\partial t = -\partial Q/\partial z - \partial I/\partial z$, where Q is the vertical turbulent kinematic heat flux (i.e., the heat flux divided by ρc_p). In view of (2), we integrate this equation with respect to z between δ and h and obtain the equation for the heat budget in the CML:

$$(h-\delta)\frac{d\theta_m}{dt} = -Q(h) + I(\delta) - I(h), \tag{5}$$

where $Q(h) = -\Delta\theta dh/dt$ is the kinematic heat flux due to entrainment on the lower boundary of the mixed layer. The profile of the vertical turbulent kinematic heat flux in the CML has the form

$$Q(z) = I(\delta)(1-\zeta) + [I(h) + Q(h)]\zeta - I(z), \tag{6}$$

where $\zeta = (z-\delta)/(h-\delta)$ is the dimensionless vertical coordinate. In both the surface and undisturbed layers, the turbulent heat flux is equal to zero.

To obtain the entrainment equation, which describes the variations in the CML depth h with time, we introduce the convective length, velocity, and temperature scales necessary to normalize the turbulence characteristics and to formulate the similarity hypotheses.

CONVECTIVE LENGTH, VELOCITY, AND TEMPERATURE SCALES

In the studies of convective flows, the following length, velocity, and temperature scales proposed by Deardorff [12, 13] are widely used:

$$h, \; w_* = (h\beta Q_s)^{1/3}, \; \theta_* = Q_s/w_*, \tag{7}$$

where h is the depth of the lower boundary of the convective layer, $\beta = g\alpha_T$ is the buoyancy parameter (α_T is the thermal expansion coefficient and g is the gravitational acceleration), and Q_s is the kinematic heat flux through the fluid surface. The Deardorff scales are applied to the flows in which the surface buoyancy flow is the source of convective motions. These scales cannot be used to describe the convection regime considered in this work. The point is that the vertically inhomogeneous solar heating is the driving force of convection beneath the ice. Its energetics has nothing to do with the molecular heat flux at the water–ice interface.

Let us introduce the following natural length, velocity, and temperature scales, which take into account the volume radiation absorption in the CML:

$$\begin{gathered} h-\delta, \; w_R = [-(h-\delta)\beta Q_R]^{1/3}, \; \theta_R = Q_R/w_R, \\ Q_R = I(\delta) + I(h) - 2(h-\delta)^{-1}\int_\delta^h I dz, \end{gathered} \tag{8}$$

where the lower index R is used instead of an asterisk to avoid confusion with the Deardorff scales. The negative sign in the expression for the velocity scale provides positiveness of w_R (according to the state equation (1), the buoyancy parameter is the temperature function $\beta(\theta) = ga_T(\theta - \theta_r)$ and is negative at $\theta = \theta_m < \theta_r$).

By analogy with the Deardorff scales, the physical meaning of the convective scales (8) can be demonstrated. Let us consider the convective layer in which the source of motions is the surface buoyancy flow. The value $w_*^3 \equiv h\beta Q_s$ is nothing but twice the velocity of the turbulent kinetic energy (TKE) generation by the buoyancy forces in the layer of depth h. This value is the integral of the vertical buoyancy flow βQ (which describes the rate of kinetic energy generation through the potential energy in the balance equation) over the convective layer. For example, for the atmospheric convective boundary layer free from clouds, in which the vertical buoyancy flow is a linear function of height, this integral is equal to $1/2h\beta Q_s$. In a similar way, the value $-1/2(h-\delta)\beta Q_R$ is nothing but the rate of the TKE generation by a vertically inhomogeneous radiation heating in the layer of depth $h - \delta$. This can be easily established by integrating Eq. (6) for z between δ and h. In this case, the heat flux through entrainment should be neglected. As the entrainment process requires an expenditure of kinetic energy, the value of $Q(h)$ cannot serve as a measure of the TKE generation rate. The velocity scale $w'_R = (-\int_\delta^h \beta Q dz)^{1/3}$, which, in contrast to w_R, contains the heat flux through entrainment, was used in [1] to estimate the velocity of convective motions in Lake Beibin. Note that, at small values of the ratio between $Q(h)$ and Q_R, which was actually the case for Lake Beibin, the numerical values of the quantities w_R and $2^{1/3}w_R$ are close to each other. One more example of geophysical flows, when the energetic considerations lead to the velocity scale similar to w'_R, is the convection in the atmospheric boundary layer with clouds. The convective velocity scale, which takes into account a radiation cooling near the upper boundary of the CML containing stratocumulus, was used by Deardorff to analyze the results of a three-dimensional numerical simulation [14].

The convective scales (8) should be verified using the data of direct measurements of the turbulence characteristics and the results of numerical calculations on the basis of an eddy-resolving model, for example, large-eddy simulations. This problem is beyond the scope of this work. We use the proposed scales in formulating the similarity hypotheses for the vertical profiles of the CML turbulence characteristics. The entrainment equation obtained on this basis is then verified using empirical data.

ENTRAINMENT EQUATION

Let us use the turbulent energy balance equation integrated over the mixed layer depth. In the absence of a shear in the mean velocity, it has the form:

$$\frac{d}{dt}\left(\int_{\delta}^{h} e\,dz\right) = -\int_{\delta}^{h} \beta Q\,dz - F_h - \int_{\delta}^{h} \varepsilon\,dz, \qquad (9)$$

where e is the turbulent kinetic energy, ε is the turbulence dissipation, F_h is the vertical energy flux at the lower boundary of the mixed layer. In the context of the similarity theory [15], let us take a hypothesis of self-similarity of the regime of turbulence in the CML, according to which the dimensionless turbulence characteristics (obtained from the normalization with the aid of the scales of the length $h-\delta$, the velocity w_R, and the temperature θ_R) are universal functions of the dimensionless depth $\zeta = (z-\delta)/(h-\delta)$. Then, the energy and dissipation profiles can be represented in the form

$$e = w_R^2\Phi_e(\zeta),\ \ \varepsilon = (h-\delta)^{-1}w_R^3\Phi_\varepsilon(\zeta), \qquad (10)$$

where Φ_e and Φ_ε are dimensionless functions.

Energy is transferred outside the CML boundaries by the radiation of internal gravity waves into the lower stably stratified layer. The energy flux due to internal waves is proportional to the value of $N^3A^2\lambda$, where $N = (-\beta\partial\theta/\partial z)^{1/2}$ is the buoyancy frequency, and A and λ are the wave amplitude and the wavelength, respectively (see, for example, [16]). According to [17–19], let us assume that A and λ are proportional to the entrainment layer depth and estimate the energy flux F_h by using the formula

$$F_h = \frac{1}{2}C_w\bar{N}^3\Delta h^3, \qquad (11)$$

where $\bar{N}$ is the mean value of the buoyancy frequency in the undisturbed layer $h < z \le D$, $\Delta h = h - h_0$ is the depth of the layer in which the vertical turbulent heat flux is negative, h_0 is the depth at which this flux is equal to zero, and C_w is a dimensionless constant. As the buoyancy frequency varies with depth for $z > h$, the value of N (averaged over the undisturbed layer) is taken as a typical value. The value of Δh is a rough value of the entrainment layer depth. The depth $z = h_0$ at which the vertical heat flux profile crosses the ordinate axis (i.e., $Q(h_0) = 0$), is determined from Eq. (6).

After substituting Eqs. (6), (10), and (11) in (9) and simple transformations, we obtain the entrainment equation in the following form:

$$(C_e + \mathrm{Ri}_\Delta)E_h - C_eE_\delta + C_w\mathrm{Ri}_N^{3/2}\left(\frac{\Delta h}{h-\delta}\right)^3 = C_\varepsilon - \frac{2}{5}C_e\mathrm{De}. \qquad (12)$$

Here, $E_h = w_R^{-1}\,dh/dt$ is the dimensionless entrainment rate; $E_\delta = w_R^{-1}\,d\delta/dt$ is the dimensionless rate of time variations in the surface layer depth; $\mathrm{Ri}_\Delta = -w_R^{-2}(h-\delta)\Delta b$ is the Richardson number based on a buoyancy jump in the entrainment layer, $\Delta b = ga_T(\theta_m + 1/2\ \Delta\theta - \theta_r)\Delta\theta$; $\mathrm{Ri}_N = w_R^{-2}(h-\delta)^2\bar{N}^2$ is the Richardson number based on the buoyancy frequency in the undisturbed layer; and $\mathrm{De} = -w_R^{-4}(h-\delta)^2d(\beta Q_R)/dt$ is the instability parameter called the Deardorff number by Zilitinkevich [17]. It is easily seen that, if the term C_eE_δ is not taken into account, Eq. (12) coincides with the entrainment equation by Zilitinkevich [17, 18] with an accuracy up to the replacement of βQ_R by βQ_s. As shown in [17, 18], this equation satisfactorily describes different regimes of convection driven by the surface buoyancy flux in the atmosphere and ocean and also under laboratory conditions. When βQ_R is replaced by βQ_s, the velocity scale w_R is transformed into the Deardorff velocity scale w_*, and the ratio $\Delta h/(h-\delta)$ is expressed, according to the known geometric formula, through the entrainment coefficient A. This latter is usually determined as the ratio (taken with an opposite sign) of the buoyancy flux through entrainment $\beta Q(h)$ to the buoyancy flux through the fluid surface βQ_s. In the case of convection driven by a vertically inhomogeneous heating, it is logical to determine the entrainment coefficient as $A = -\beta Q(h)/\beta Q_R$.

Equation (12) contains three dimensionless constants $C_\varepsilon = 1 - 2\int_0^1\Phi_\varepsilon(\zeta)d\zeta$, $C_e = \frac{10}{3}\int_0^1\Phi_e(\zeta)d\zeta$, and C_w, which must be determined in comparing the results of calculations of different entrainment characteristics with experimental data. There is also another way of determining C_ε and C_e—a direct estimate of the integrals of the functions Φ_ε and Φ_e. The estimates $C_\varepsilon = 0.2$, $C_e = 0.8$, and $C_w = 0.01$ were obtained in [17–19] from laboratory, atmospheric, and oceanic data; in this case, C_ε and C_e are determined in the two ways. We shall use these values as well.

COMPARISON OF MODEL CALCULATIONS WITH EMPIRICAL DATA AND DISCUSSION OF THE RESULTS

The vertical temperature profiles calculated on the basis of the model proposed were compared to the data obtained from the measurements taken in Lake

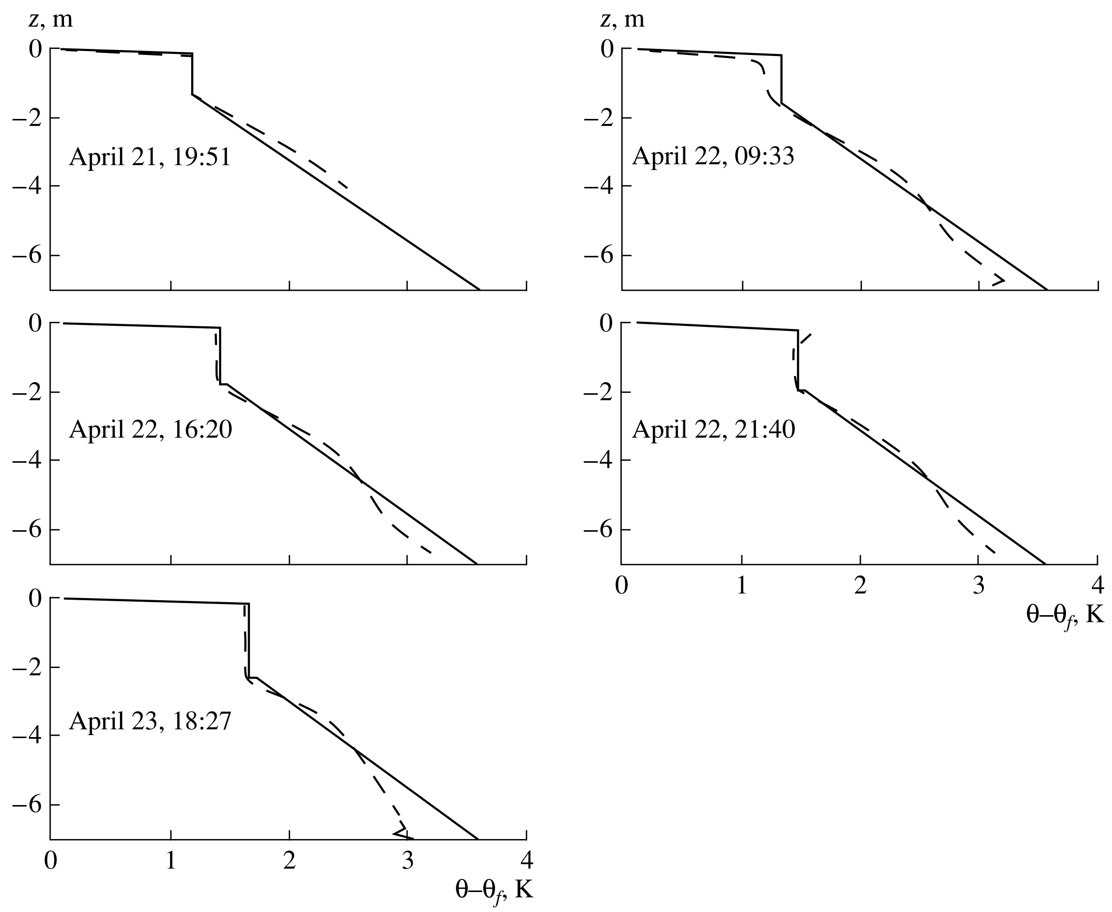

Fig. 1. Sequential temperature profiles for Lake Vendyurskoe within the period of penetrative convection. Solid lines correspond to the profiles calculated on the basis of the model proposed, and dashed lines correspond to the observational data. Dates and local time are indicated in the left parts of the figures. The observation point depth is 7.7 m.

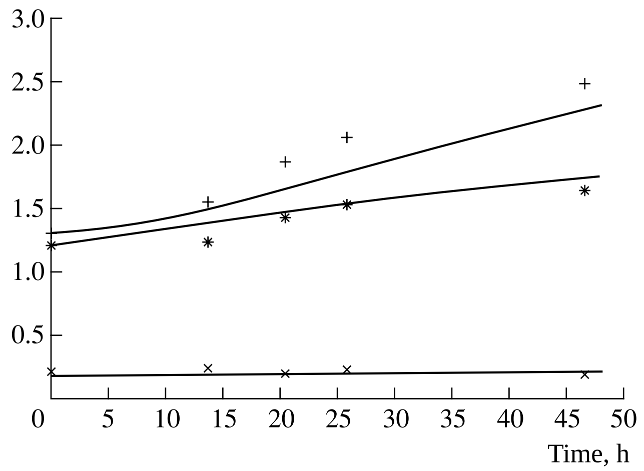

Fig. 2. Time variations in the CML lower boundary depth h, water temperature θ_m in the CML, and the surface layer depth δ for Lake Vendyurskoe during the period of penetrative convection. Curves correspond to the model data, and the symbols (pluses, asterisks, and crosses) correspond to the observational data on mixed layer depth, temperature, and surface layer depth, respectively.

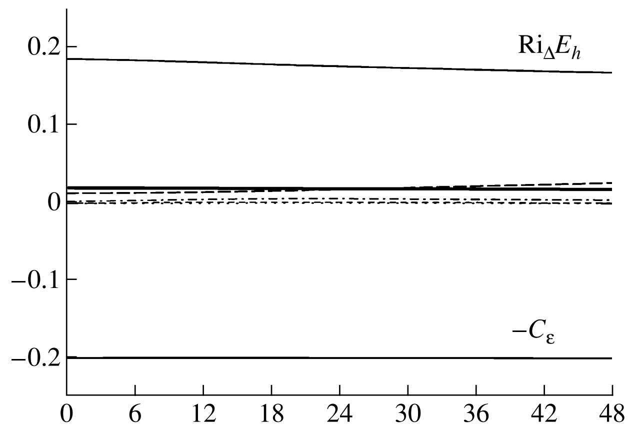

Fig. 3. Terms of the entrainment equation (12) as time functions. Solid lines at the bottom and at the top denote $-C_\varepsilon$ and $\mathrm{Ri}_\Delta E_h$, respectively. The rest of the lines (from bottom to top): short-dash line corresponds to $2/5C_eDe$, dotted line corresponds to $-C_eE_\delta$, dot-and-dash line corresponds to C_eE_h, dashed line corresponds to $C_w\mathrm{Ri}_N^{3/2}\,[\Delta h/(h-\delta)]^3$, and heavy line corresponds to R.

Vendyurskoe, Karelia, in April 1995. A detailed description of the measurements and a full data set are given in [3, 4, 9]. Let us assume that the short-wave solar radiation absorption is described by the exponential law $I(t, z) = I_s(t) \times \sum_{i=1}^{n} a_i \exp(-\gamma_i z)$, where I_s is the radiation flux at the water–ice interface, a_i are the portions of the total radiation flux for n different spectral ranges, and γ_i are the absorption coefficients. Within the framework of the two-band approximation [3] on the basis of the data of direct measurements of radiation flux in Lake Vendyurskoe, the following estimates were obtained: $a_1 = a_2 = 0.5$, $\gamma_1 = 2.7$, and $\gamma_2 = 0.7$. The constant value of $I_s = 8 \times 10^{-6}$ K m s^{-1} characteristic of the observation period was used in the calculations. The vertical temperature profile evolution during a two-day period was calculated using the estimates proposed above and a linear approximation of the initial temperature profile in the undisturbed layer. As is seen from Figs. 1 and 2, the calculation results are in good agreement with the observational data. It should be stressed that the values of the constants C_ε, C_e, and C_w in the entrainment equation and also the values of the coefficients a_1, a_2, γ_1, and γ_2 in the law of short-wave solar radiation absorption were independently determined and were not fitted to the calculated values. It should also be noted that there is a good agreement between the calculated and experimental values of the surface layer depth δ. The surface layer model, despite its simplified character, describes the surface layer depth very accurately. It is needless to say that a consideration of the diurnal cycle of solar radiation flux will require some changes in the surface layer model; in particular, the assumption that the heat transfer regime is quasi-stationary must be abandoned.

The time variations in different terms of Eq. (12) are given in Fig. 3. The basic terms in the entrainment equation are C_ε and $\mathrm{Ri}_\Delta E_h$. The term $C_w \mathrm{Ri}_N^{3/2} [\Delta h/(h - \delta)]^3$ is considerably smaller than the two main terms, but not negligibly small. The rest of the terms are at least 1.5 orders of magnitude smaller. Thus, if the diurnal cycle of solar radiation flux is not taken into account, the entrainment regime characteristic of the convection in an ice-covered lake proves to be very similar to that characteristic of the "typical" conditions in the atmospheric boundary layer (ABL). In a first approximation, the two regimes are described by the simplest equation $\mathrm{Ri}_\Delta E_h = C_\varepsilon$; i.e., the constant entrainment coefficient $A = 0.2$, where $A = -Q(h)/Q_s$ for the ABL and $A = -\beta Q(h)/\beta Q_R$ for convection beneath the ice. Such an entrainment regime is accurately simulated in laboratory experiments with a two-layered fluid [18]. Energy transfer by internal waves outside the CML results in a decrease of the rate of deepening the mixed layer and, thus, in a decrease of A. Since the entrainment equation (12) is universal, a successful description of one or another convection regimes depends primarily on a choice of the scales of length, velocity, and heat flux (temperature). Good agreement between the theoretical predictions and the empirical values given in Figs. 1 and 2 suggests that the scales (8) are appropriate for the description of convection driven by a vertically inhomogeneous heating in an ice-covered lake.

Farmer [1] described the entrainment efficiency by the ratio $R = -\int_{h_0}^{h} \beta Q dz \,/\, \int_{\delta}^{h_0} \beta Q dz$, which was called the "energetic" entrainment coefficient by Petrov and Sutyrin [2]. In the case of convection driven by a surface buoyancy flux, when βQ is a linear function of z, $R = A^2$. The time variations in R are shown in Fig. 3 (heavy line). For a period of calculations, the mean value of R is 0.019. This estimate is within the limits of the scatter in the empirical estimates of R between 0.003 and 0.113 with a mean value of 0.036 obtained from the measurements in Lake Beibin [1]. (Note that, in [1], to calculate R, an approximate formula slightly overestimating R was used.) The calculation of CML evolution without considering energy transfer by internal waves outside the mixed layer (i.e., at $C_w = 0$) yielded $R = 0.024$, which corresponds to $A = 0.2$.

The terms $C_e E_\delta$ and $\frac{2}{5} C_e \mathrm{De}$ in (12), which are negligibly small in our calculations, may prove to be important under the conditions of a strong instability of the solar radiation flux, for example, in considering the diurnal cycle of I_s. The term $C_e E_h$, which is also small in our calculations, is the basic term in Eq. (12) (along with C_ε) in the regime of the CML deepening against the background of neutral stratification in the undisturbed region. In this case, if E_δ and De are small, Eq. (12) is reduced to the simple relation $E_h = C_\varepsilon/C_e$. Petrov and Sutyrin showed that such a regime of CML deepening is realized in an ice-covered lake during morning hours [2]. This regime becomes steady after the sunrise and persists until the lower boundary of the convective zone reaches the jump layer formed the day before. Note also that the term $C_e E_h$ is not small if a zero jump in temperature at the outer boundary of the mixed layer (in our calculations $\Delta\theta = 0.1$ K at $t = 0$) is given as an initial condition. This term is important at small times until the solution of the problem reaches the "equilibrium" regime and "forgets" the initial conditions.

The deepening of the mixed layer in stably stratified fluids that is not accompanied by a temperature jump at its outer boundary is one more limiting case. This so-called encroachment regime was first considered by Zubov [20]. Based on the equation for the CML depth with $R = 0$, a model of convection in an ice-covered lake was considered by Bengtsson [7]. As was shown by Farmer [1], whose entrainment equation includes the Zubov asymptotics as one of the limiting cases, the model with $R = 0$ underestimates the rate of growth of the mixed layer depth.

CONCLUSIONS

The regime of penetrative convection driven by a vertically inhomogeneous radiation heating is considered. A similar convection is observed in ice-covered freshwater lakes, where the water temperature is below the maximum-density temperature during late spring. The convective length, velocity, and temperature scales are introduced, which are suitable for the description of convection driven by a vertically inhomogeneous radiation heating. On the basis of these scales and the similarity hypotheses for the vertical temperature profiles and turbulence characteristics in the CML, a simple model of the convection regime under consideration is proposed. The results of calculations based on this model are in good agreement with the observational data.

The proposed convective scales must be carefully verified using the results of direct measurements of the turbulence characteristics and also the results of numerical calculations on the basis of an eddy-resolving model (for example, large-eddy simulation). Recall that the results of a direct numerical simulation of the convective planetary boundary layer and Rayleigh convection helped Deardorff to formulate the convective scales that came to be classical.

In conclusion, let us note that a convection regime similar to that considered in this paper occurs in freshwater pools on the melting sea ice. Since the water in such pools is actually fresh and its temperature is below the temperature of maximum density, vertically inhomogeneous solar heating will cause convective motions just as it occurs in freshwater lakes. These two convection regimes differ from each other by their boundary conditions. The water surface temperature in freshwater pools (i.e., temperature at the water–air interface) varies with time, while in lakes the temperature at the water–ice interface is fixed at the freezing point. The near-bottom temperature varies with time in lakes and is equal to the temperature of freezing in freshwater pools. From the practical point of view, it is very important to know how to calculate the vertical convective heat transfer in the freshwater pools. Convective motions intensify heat transfer towards the ice surface and, thus, affect the process of ice melting.

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